647510 **MSC2MatC203x** **Seat No :________**

M.Sc. (Maths) Semester - 2 (*CBCS*) Examination
May/June -2021 [NEW COURSE]
Topology – 2 (CORE)

Time: 1:30 Hours **Marks: 42**

Instructions:

1. Figure to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any three of the following questions.

Q.1 (a)		**4 Marks**
(1)	Show that a subspace of a Hausdorff space is Hausdorff.	2
(2)	Let X be a Hausdorff space, and let (x_n) be a sequence in X. Show that the limit of (x_n), if exits, is unique.	2
Q.1 (b)	**Answer any two question out of three.**	**10 Marks**
(1)	Let X be a space satisfying the T_1- axiom, let A be a subset of X, and let $x \in X$. Show that x is a limit point of A if and only if every neighborhood of x contains infinitely many points of A.	5
(2)	Let X be a topological space. Show that X is a Hausdorff space if and only if $\Delta = \{(x,x) : x \in X\}$ is closed in $X \times X$.	5
(3)	Show the T_1- axiom is equivalent to the condition that for each pair of points of X, each has a neighborhood not containing the other.	5
Q.2 (a)		**4 Marks**
(1)	Show that a closed subspace of a normal space is normal.	2
(2)	Show that the space $\mathbb{R}_\ell$ is regular.	2
Q.2 (b)	**Answer any two question out of three.**	**10 Marks**
(1)	Let $K = \{1, \frac{1}{2}, \frac{1}{3}, \ldots\}$, let $B = \{(a,b) \setminus K : a, b \in \mathbb{R}, a < b\} \cup \{(a,b) : a, b \in \mathbb{R}, a < b\}$, and let τ be the topology on $\mathbb{R}$ generated by B. Show that $(\mathbb{R}, \tau)$ is Hausdorff but not regular.	5
(2)	Show that every regular space with a countable basis is normal.	5
(3)	Show that every metric space (X, d) with the metric topology is completely regular.	5
Q.3 (a)		**4 Marks**
(1)	Show that $\mathbb{R}$ with the standard topology is not compact.	2
(2)	Let X be a locally compact Hausdorff space, and let A be a subspace of X. If A is closed in X, then show that A is locally compact.	2
Q.3 (b)	**Answer any two question out of three.**	**10 Marks**
(1)	Let X be a topological space. Show that X is compact if and only if for every collection $\mathfrak{C}$ of closed sets in X having the finite intersection property, the intersection $\bigcap_{C \in \mathfrak{C}} C$ of all the elements of $\mathfrak{C}$ is nonempty.	5
(2)	Let A and B be disjoint compact subspaces of the Hausdorff space X. Show that there exist disjoint open sets U and V containing A and B, respectively.	5
(3)	Show that compactness implies limit point compactness, but not conversely.	5
Q.4 (a)		**4 Marks**
(1)	Let (X, d) be a metric space, $x \in X$ and $\emptyset \neq A \subset X$. If $d(x, A) = 0$, then show that $x \in \bar{A}$.	2
(2)	State the Extreme value theorem.	2

Q.4 (b)		**Answer any two question out of three.**	**10 Marks**
	(1)	Let X be a metrizable topological space. If X is sequentially compact, then show that X is compact.	5
	(2)	Show that a metric space (X, d) is compact if and only if it is complete and totally bounded.	5
	(3)	Let $\mathcal{A}$ be an open covering of the metric space (X, d). If X is compact, then show that there is a $\delta > 0$ such that for each subset of X having diameter less than δ, there exists an element of $\mathcal{A}$ containing it.	5
Q.5 (a)			**4 Marks**
	(1)	Show that $(-1,1)$ with the usual metric is not complete.	2
	(2)	Define completion of a metric space.	2
Q.5 (b)		**Answer any two question out of three.**	**10 Marks**
	(1)	Let (X, d) be a metric space. Show that there is an isometric imbedding of X into a complete metric space.	5
	(2)	If the space Y is complete in the metric d, then show that the space Y^J is complete in the uniform metric $\bar{\rho}$ corresponding to d.	5
	(3)	Let X be a topological space and let (Y, d) be a metric space. Show that the set $C(X, Y)$ of continuous functions is a complete metric space under the uniform metric.	5

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